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ABSTRACT

This study investigates relationships between differences in students' achievement and differences obtained by a technique held to yield a reasonably valid representation of students' memory structures of a particular subject matter. Also investigated was the nature of the flow of information between curriculum materials, teachers, and students. This was achieved by a comparison of the digraphs of content structure and the graphs constructed by the teachers and the students. The subjects were five mathematics teachers (grades 7-10) and their six classes (student n = 109). The content used was a curriculum package on operational systems which involved one lecture and a seven-page booklet for students to study. Students and teachers on the average reflected the content structure of the subject in their constructed graphs. There were differences in achievement corresponding to the correctness of students' graphs of the content structure, but these differences also reflected variations among the teachers' graphs. (JP)

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REPRESENTATION OF SUBJECT MATTER IN TEACHERS' AND STUDENTS' MEMORIES

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In recent years, mathematical curricula have tended to move away from emphasizing rote learning and computational skills and toward greater emphasis on teaching subject matter structure (Bruner, 1960; Klepfer, 1971; Schwab, 1962; Wilson, 1971). This shift in emphasis has been accompanied by a variety of justifications, including claims that instruction oriented toward learning the structure of the curricular material leads to a fuller understanding of the subject matter, which in turn results in enhanced achievement, increased problem-solving ability, and transfer to new situations, as well as enhanced retention and increased student enthusiasm for the subject matter.

Although such expected outcomes of the structure-teaching approach are admirable and of considerable educational value, there is not a great deal of empirical evidence to support the claim that these outcomes do actually occur (but see Greeno, 1972; McKenzie, 1972; Mayer and Greeno, 1972; Shavelson, 1972). This study is an attempt to investigate relations between differences in student achievement and differences obtained by a technique held to yield a reasonably valid representation of students' memory structures of a particular subject matter.

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The differences between an achievement test and a representation of cognitive structure should be made explicit in regard to the kinds of data obtained. An achievement test usually provides evidence for such student variables as precision of discrimination between concepts, accuracy of conceptual definitions, and appropriateness of conceptual application. However, a valid representation of a student's cognitive structure should indicate how a student associates key curriculum concepts, which clusters of concepts are considered relatively important, and the degree of inclusiveness of conceptual clusters. Such a representation could then provide the reference for data bearing on assumed correlates or consequences of attaining an appropriate structure. As an example of such a correlate consider the question: Do students with memory structures corresponding closely to the subject matter structure achieve at a higher level than students evidencing a lower degree of such correspondence? A valid representation of cognitive structure could also be employed, in conjunction with representations of content structure, to analyze results of the flow of information between curriculum materials, teachers, and students.

METHOD

Subjects: The subjects were five mathematics teachers and their six classes (student n = 109). The teachers were completing a fifth-year teacher training program leading to state certification as mathematics teachers. The students ranged from 7th to 10th grade level, and represented a range of socio-economic status, as well as a range of mathematical proficiency.

Instructional Material: The primary content of this study was a curriculum package on operational systems. An operational system is defined as a set together with a binary operation on the set. An operational system may possess any, or none, of the following fundamental properties: associativity, commutativity, identity element, and roundness. This content was selected because operational systems is a mathematical structure including concepts that can be hierarchically arranged, and because concepts embedded in the operational systems curriculum are fundamental to many other mathematical structures and are, therefore, mathematically significant.

Employing material developed by the Secondary School Mathematics Curriculum Improvement Study (1967) and the Comprehensive School Mathematics Project (1970), two mathematics curriculum specialists (Branca and Pelavin) developed the actual operational systems curriculum package, which consisted of an hour lecture and a seven-page text. The text gave several examples of operational systems and summarized the material discussed in the lecture. From this package, the following twelve concepts were identified as crucial: associativity, binary operation, commutativity, element, finite/infinite, fundamental properties, identity element, inverse, operational system, ordered pair, roundness, and set.

Figure 1 presents a hierarchical graph representing the relational structure, or organization, of these key concepts that was obtained from the curriculum developers. They considered this representation to be correct and meaningful.

Instrumentation: The curriculum developers', teachers' and students' cognitive structures were examined by first administering a linear-graph-building task (Fillenbaum and Rapoport, 1971). In this task, subjects were provided a list of the key concepts. They then selected what they felt were the two most similar concepts and connected them with a line (which they labeled "1"), thereby forming a cluster. The remaining concepts were similarly associated, either among themselves or with clusters already established. Then a proximity matrix was formed, using the sums of the numbers on the lines connecting each concept with every other concept as an index of the intra-conceptual distance. This proximity matrix served as input to Johnson's (1967) hierarchical clustering (HICLUS) program, which yielded a representation of the hierarchical structure underlying the linear graph.

Two parallel forms of an achievement test were constructed for this research project. One form was administered as a measure of pre-treatment achievement, and the alternate form was used as a measure of post-treatment achievement. Each form contained items covering the 12 key concepts, with the items spread across three levels: (a) knowledge of definitions of key concepts, (b) familiar problems, and (c) unfamiliar problems.

The content structure of the curriculum was analyzed by a digraph method (Shavelson, 1972, 1974), and yielded the HICLUS representation presented in Figure 2.

Procedures: The curriculum developers held an instructional session for the teachers, at which time the lecture developed on the operational systems curriculum was given. The seven-page text was also

given to the teachers at this time. After the session, one form of the achievement test was administered to the teachers. Subsequently, the teachers prepared lesson plans based on the material covered in the session with the curriculum developers and in the seven-page text. Next, the teachers administered the graph-building task and one form of the achievement test to their students, spent about three class periods teaching operational systems, and then re-administered the graph test along with the alternate form of the achievement test. About a week later, the graph-building task was administered to the teachers.

RESULTS

The achievement scores obtained from the teachers were essentially at ceiling, supporting the conclusion that they understood the operational systems material according to this manner of measureing comprehension.

Figure 3 presents the HICLUS hierarchical structure resulting from the combined dissimilarity matrices obtained from each teacher's linear graph. This hierarchical graph is almost identical to those in Figures 1 and 2, indicating that the teachers' cognitive structures were, on the average, good and correct. The flow of information from the curriculum materials to the teachers underwent no significant distortion observable in this grouped data.

Figure 4 presents the HICLUS structure obtained from the combined linear graphs of all students prior to instruction, and Figure 5 presents the post-instructional structure for all students. Although very fine-grained comparisons between graphs are probably

not justifiable, it seems clear that there are some reasonably large differences between the two graphs that give evidence of change in the direction of greater congruence with the "ideal" structure represented in figures 1,2, and 3. For example, at the pre-instructional stage, the students do not connect the concept of identity element with the roundness and inverse concepts. This problem is not manifest in the post-instructional graph. Also, at the pre-instructional stage, roundness and inverse enter very late into the structure. Such a result would occur if these concepts were not connected with the other concepts until the final steps of the graph-building task. Students not understanding these concepts would be expected to behave in this manner. At the post-instructional stage, however, these concepts form subclusters at a lower level, i.e., they are collected into sub-clusters notably earlier than the point at which the sub-clusters are interconnected. As well, the post-instructional graph indicates that the students, in general, have learned to connect identity element into a sub-cluster with roundness and inverse. Also, the post-instructional clusters seem to be formed more directly and precisely. However, the post-instructional structure also indicates that, for the total group, the erroneous connection of the binary operation-operational system pair seems to persist. Although interrelating operational system with the fundamental properties might be justifiable, since "operational system" is a somewhat ambiguous stimulus as an isolated phrase, binary operation seems clearly more accurately considered as a defining characteristic of operational

systems, rather than as a fundamental property.

The next step in this investigation was to split the students on the basis of their achievement scores. Within each class, the students were classified as above or below that class's median level of achievement on the pretest. The same procedure was followed for classification on the basis of the posttest scores. This within-class designation of students as "high" or "low" was employed, as opposed to a between-class designation based on pooling the achievement scores of the total groups, in an attempt to partially control for disproportionate "class effects" attributable to between-class differences such as level of general ability, aptitude, age, differences in teaching effectiveness, etc. Although unequal class size lessens the effectiveness of this technique, the class effect is at least composed of within-class components in the same proportion in both the high- and the low-achievement groups.

Achievement on the pre-test was at a generally low level. Out of 18 items, class averages hovered around 7. Visual inspection of the HICLUS structures resulting from the pre-test split indicated that the two sub-groups were essentially the same, i.e., naive vis-a-vis operational systems.[1]

The achievement post-test contained 29 items, and class averages were around 18. This higher level of achievement corroborates the indication, from comparing Figures 4 and 5, that students did appear to benefit from the instruction. Figures 6 and 7 indicate meaningful differences between the high- and low-achievement groups on the posttest. According to these two structures, the major difference

between the two groups centers on the manner in which the conceptual trio of binary operation, operational system, and ordered pair is interrelated with the other concepts. In Figure 6, the error persists of connecting binary operation with the sub-cluster of fundamental properties, whereas in Figure 7, binary operation is correctly connected with the concept of ordered pair, and is located within the sub-cluster of defining characteristics. This difference between Figures 6 and 7 involves a distinction of some importance, and confusion on this matter might be expected to impair mathematical achievement.

One way to analyze the flow of information between the teachers and students of this study is to compare each teacher's graph with their class's graph. These graphs are presented in Figures 8 through 12 Comparisons show that an unusual or improper placement of concepts in a teacher's graph frequently corresponds to a similarly improper conceptual organization in the class's graph. For example, in Figure 8, teacher 10 has improperly located the identity concept with the cluster of concepts related to the topic of set, rather than with those concepts concerned with fundamental properties; this same peculiarity is reflected by his class's graph. Another example is found by examining Figure 9. In this figure, both teacher and class locate binary operation and operational system as a grouping within the fundamental properties cluster. Additional examples are easily located, indicating that just as the average teachers' structure corresponded closely to that of the curriculum makers, so does the average of a class's structure correspond closely to that of the teacher.

Additional research is planned on the topic of the present study. Some research is suggested by the limitations and results of this study in particular, as well as from the limitations of correlational studies in general. For example, even given a more refined measure of conceptual interrelations, there remains the problem of developing a meaningful index of the degree to which a subject's structure approaches a particular criterion structure. Such a measure would enable subjects to be split on the structure dimension, rather than on the achievement dimension, as well as provide the basis for deriving a correlation between cognitive structure and achievement. Also, tighter experimental control, including random assignment of students to teachers and better control of the instructional process, would be highly desirable aspects of future investigation. Finally, the results obtained thus far indicate that the experimental manipulation of the structure variable may be the most informative next step in this area. By mapping variables relating to structure onto a design incorporating experimental manipulation, many of the issues and implications of the present study can be more rigorously investigated. A study is currently being planned in which experimental groups will vary according to whether or not they are taught a structure. Such a design can provide data regarding causal relations between the (degree of) acquisition of a structure and student performance on achievement, retention, and transfer tasks, hypothesized interactions involving student aptitudes, and the generalizability of the results of the present study.

FOOTNOTE

1. A limitation of the HICLUS method should be mentioned, and it seems appropriate to do so at this point. The graph representation does not give an indication of the extent to which the branching points reflect extensive or marginal agreement between subjects. Neither is there an indication of the alternative patterns among which the program had to decide, nor of the relative strength of these alternatives. The HICLUS program might perhaps be adaptable to yield information on these matters. Other, perhaps non-hierarchical, clustering techniques currently being developed may prove to be of value for subsequent work on this problem of refining means of representing cognitive structure.

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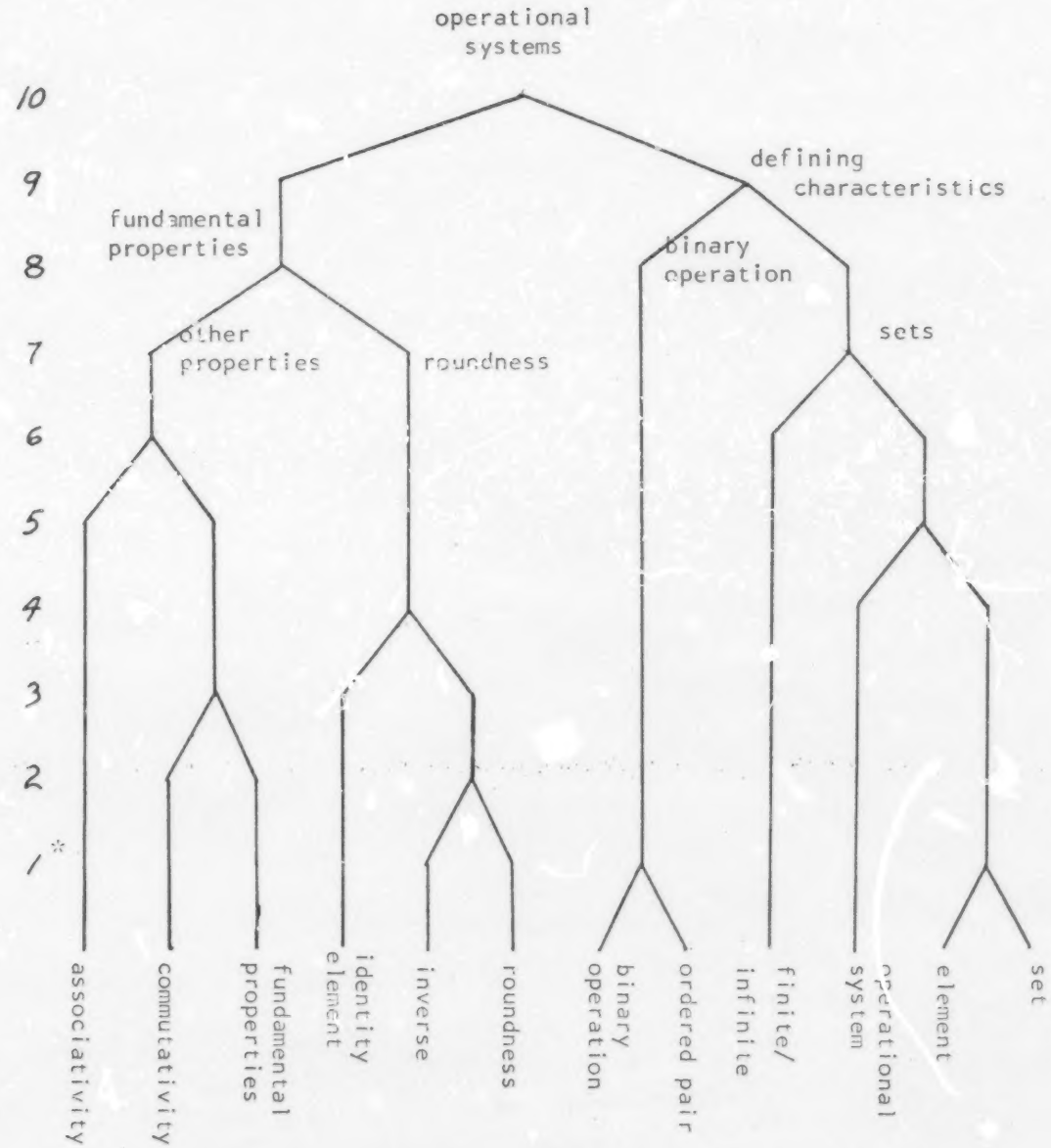


Figure 1. HICLUS representation of curriculum developers' linear graphs.

*In each figure, these numbers indicate the order in which the groupings occur. For example, in this figure, inverse and roundness constitute the second grouping, and the cluster of finite/infinite, operational system, element, and set constitute the seventh grouping, which is an identifiable sub-cluster, labeled "sets."

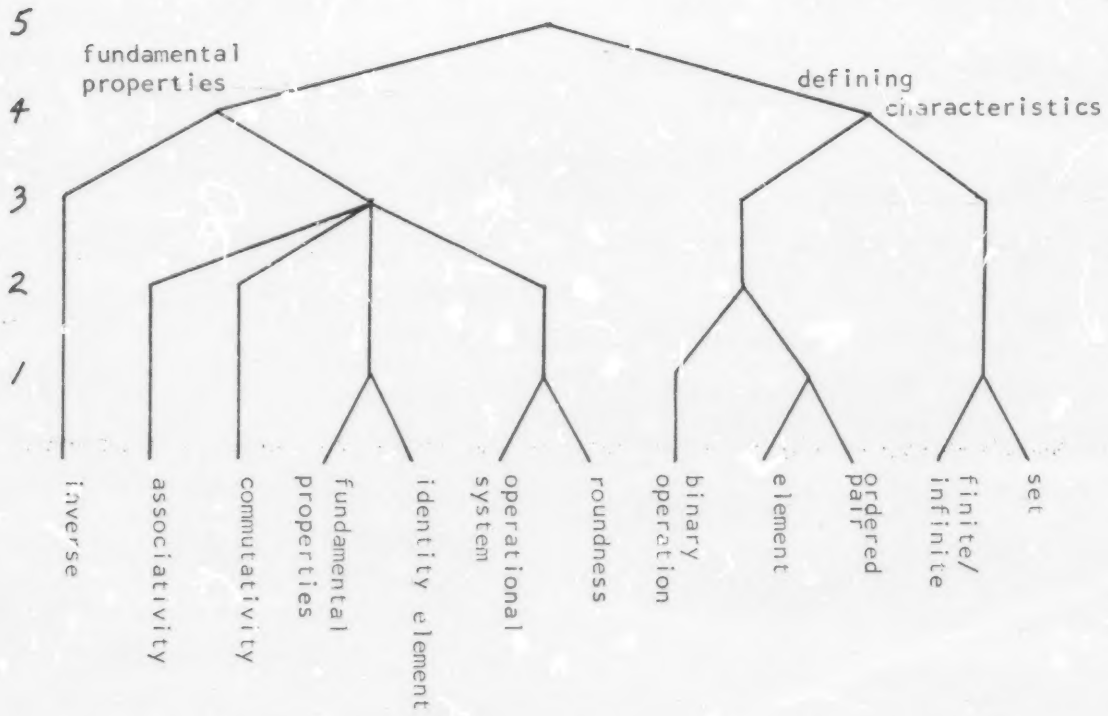


Figure 2. HICLUS representation of the digraph analysis of the content structure.

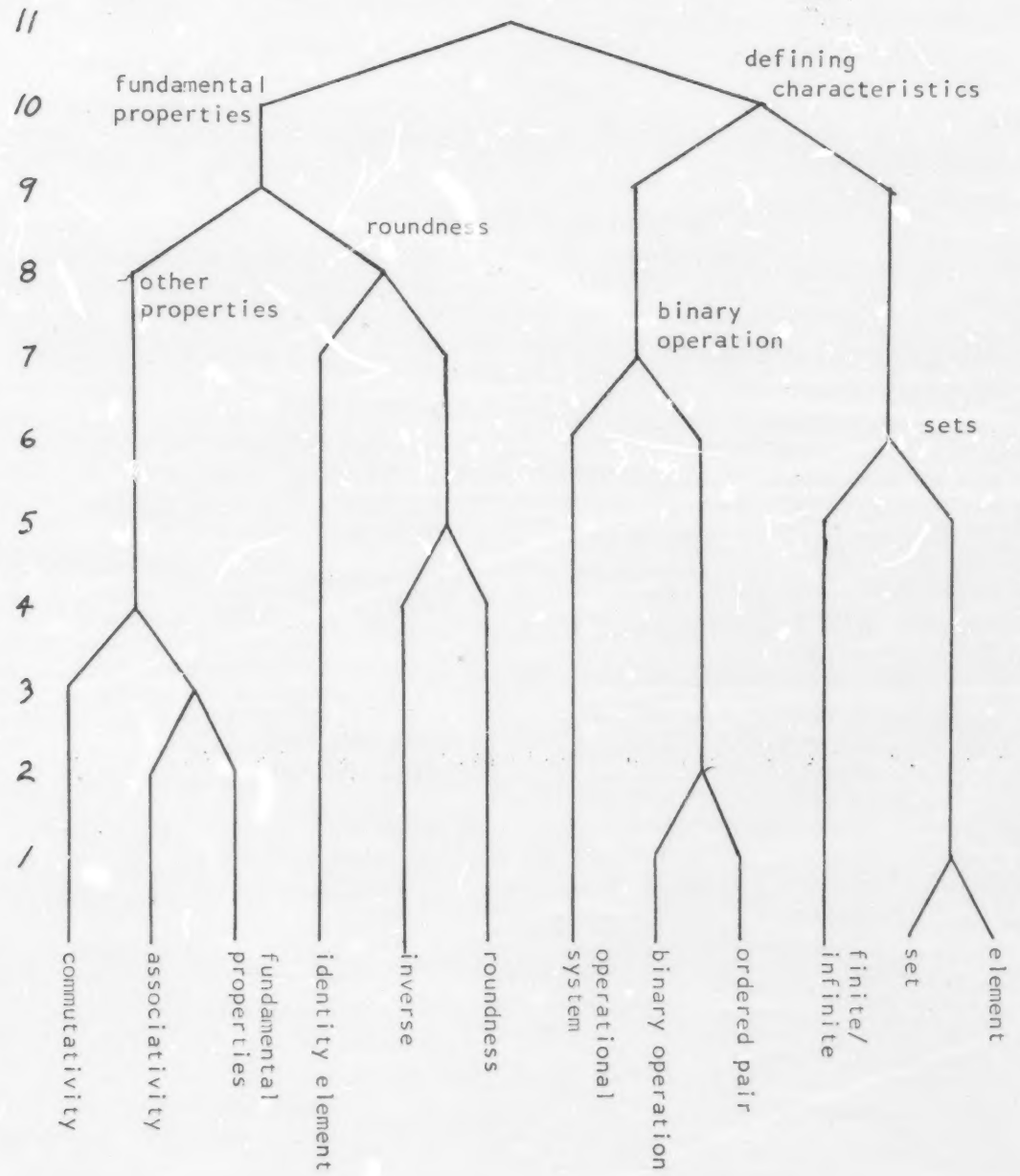


Figure 3. HICLUS representation of teachers' combined linear graphs.

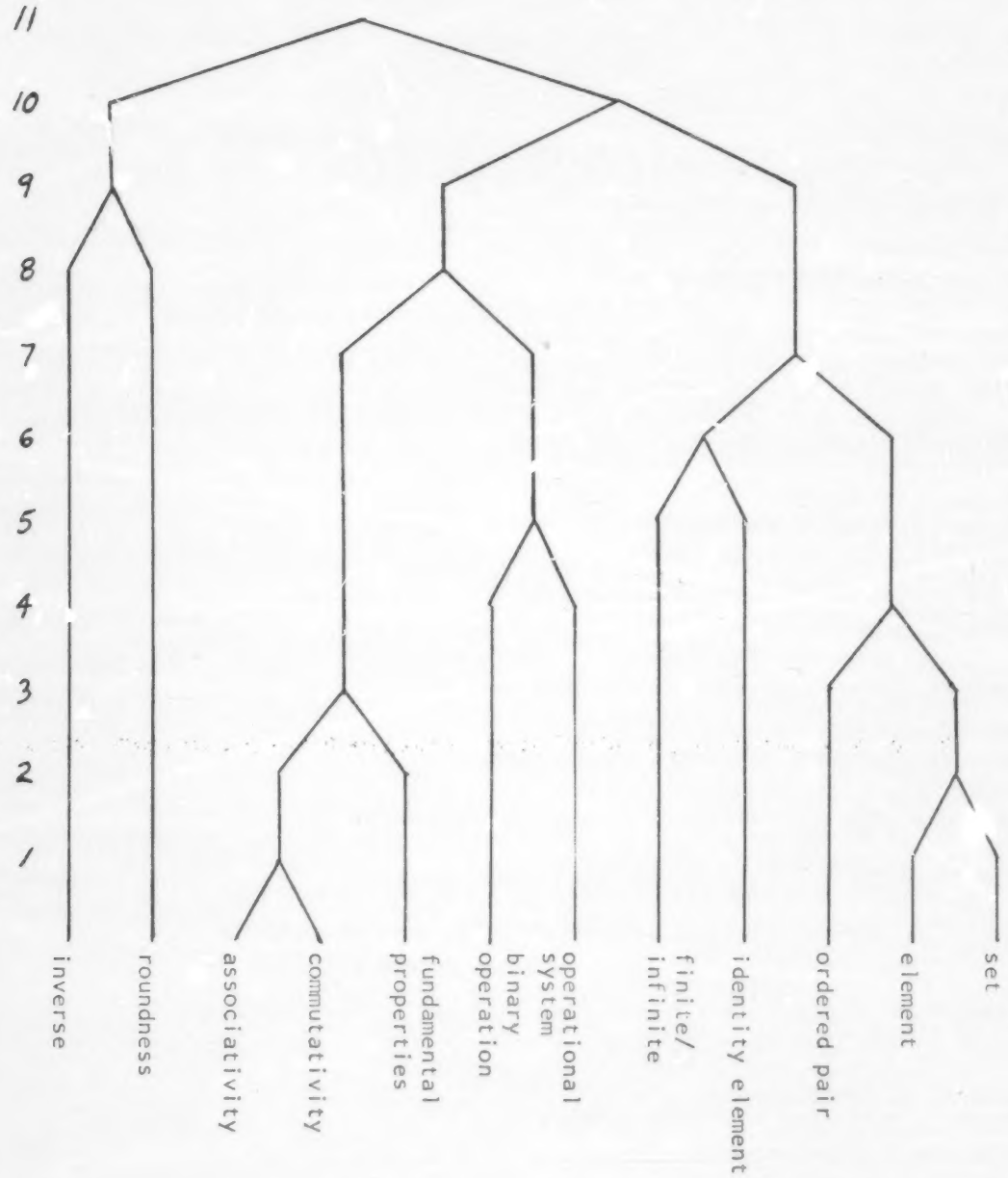


Figure 4. HICLUS representation for all students--pre-instruction.

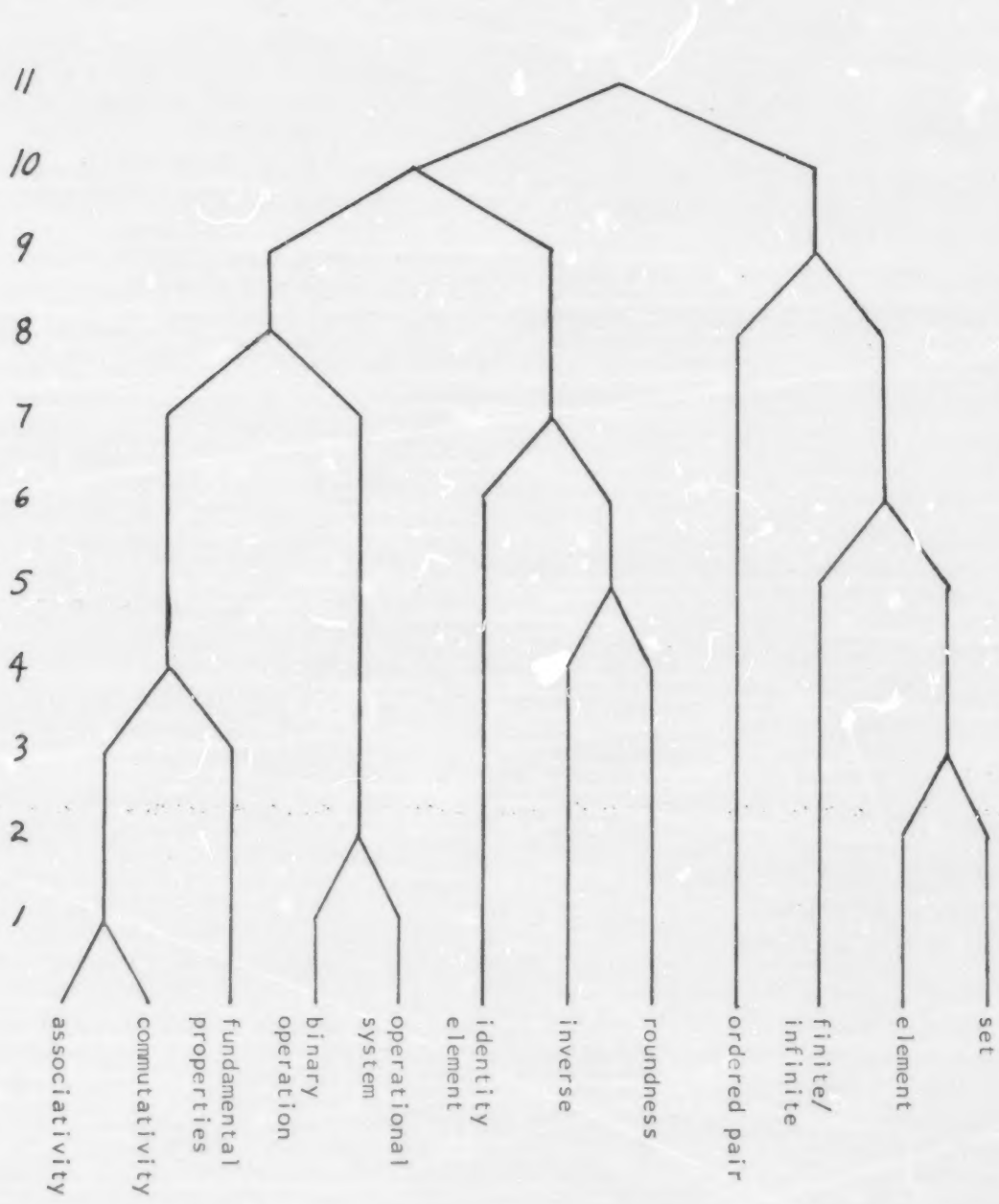


Figure 5. HICLUS representation for all students--post-instruction

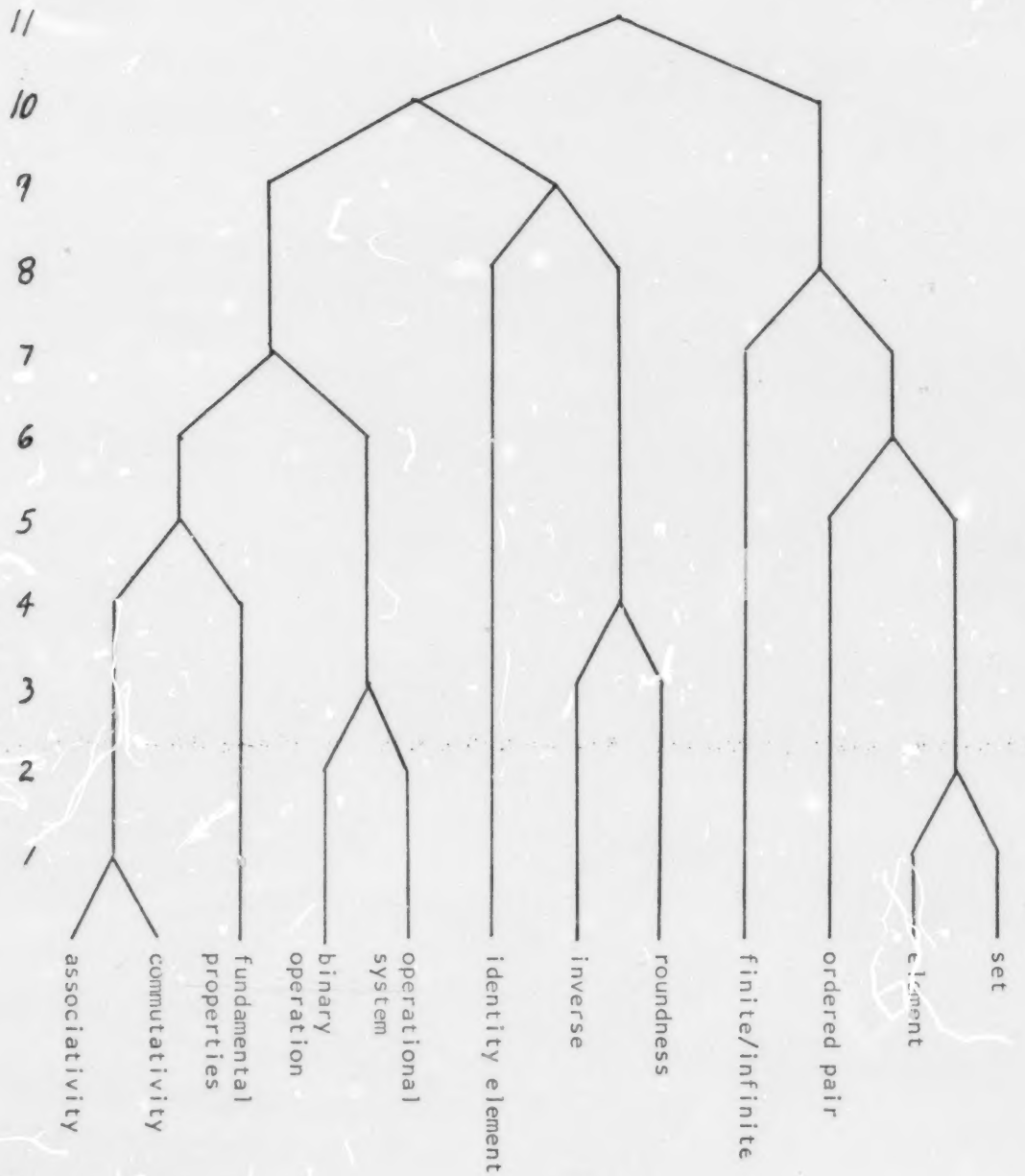


Figure 6. HICLUS representation for students below achievement median--post-instruction.

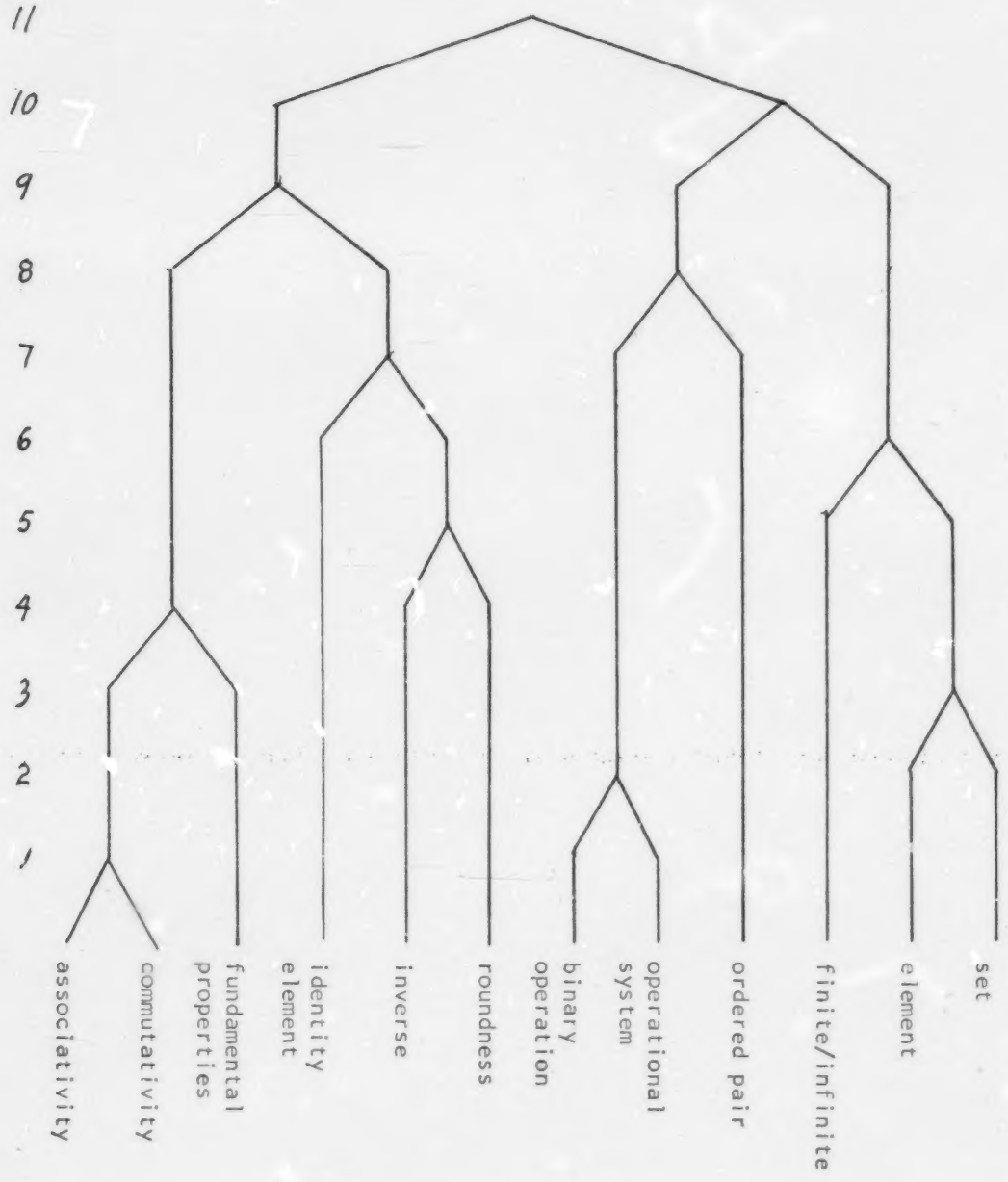
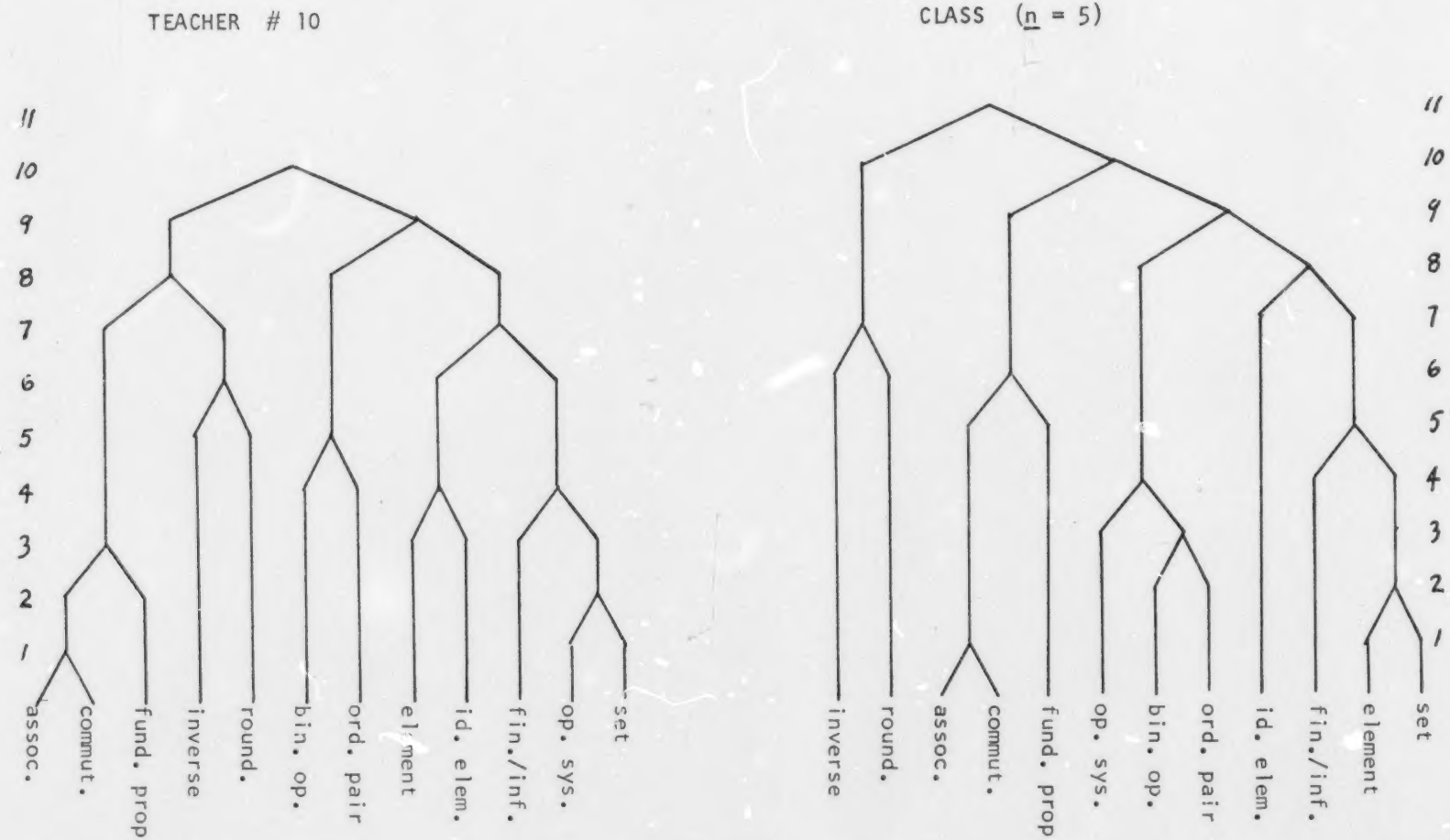


Figure 7. HICLUS representation for students above achievement median--post-instruction.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id. elem.
fin./inf.
op. sys.
set
CLASS (n = 5)
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id. elem.
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element
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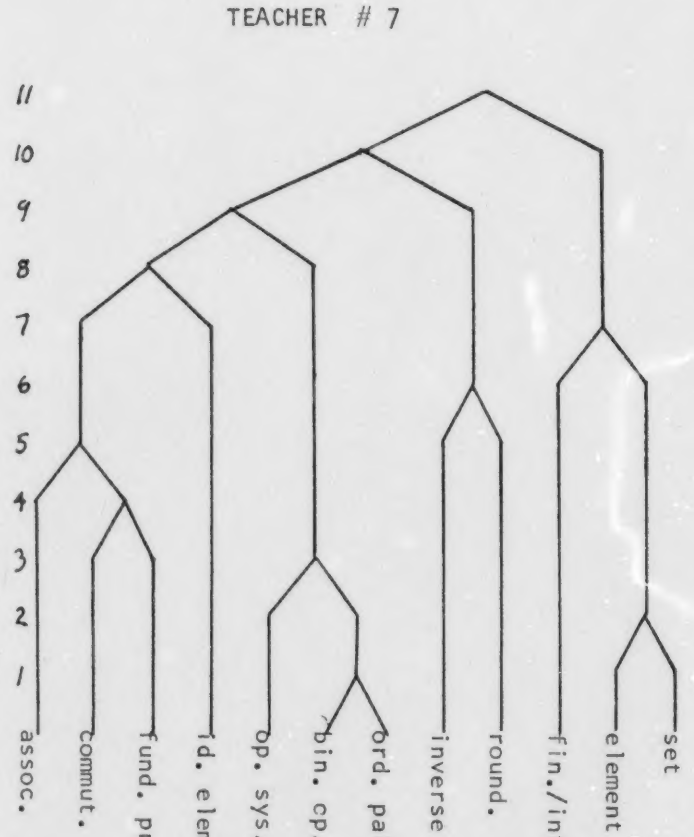
Figure 8.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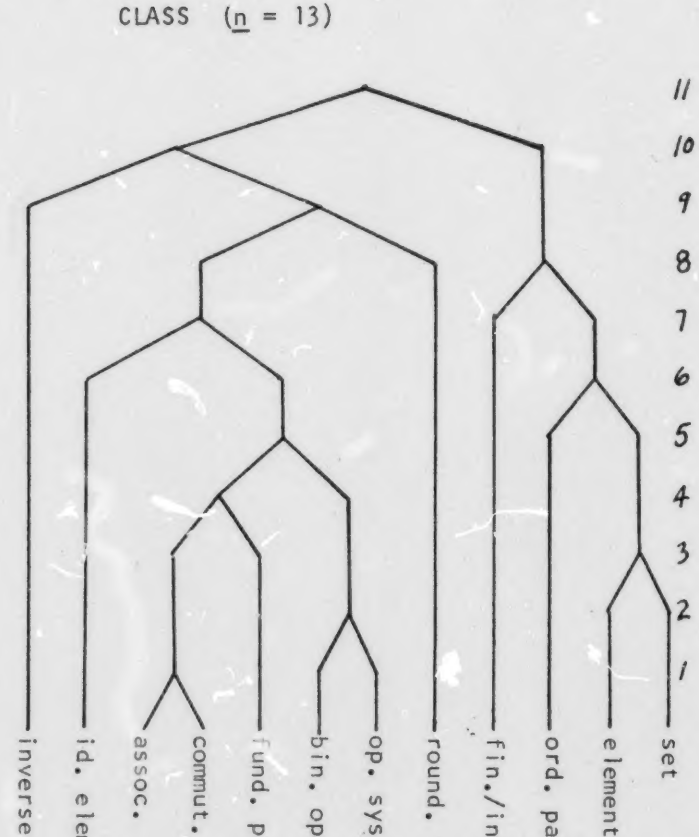
CLASS (n = 13)
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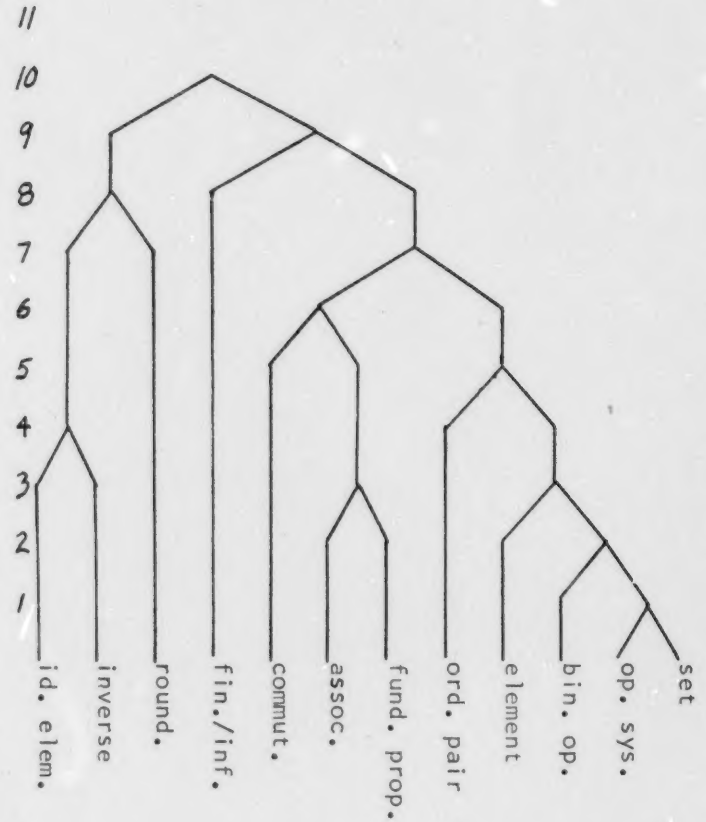
Figure 9.

TEACHER # 8



CLASS (n = 16)

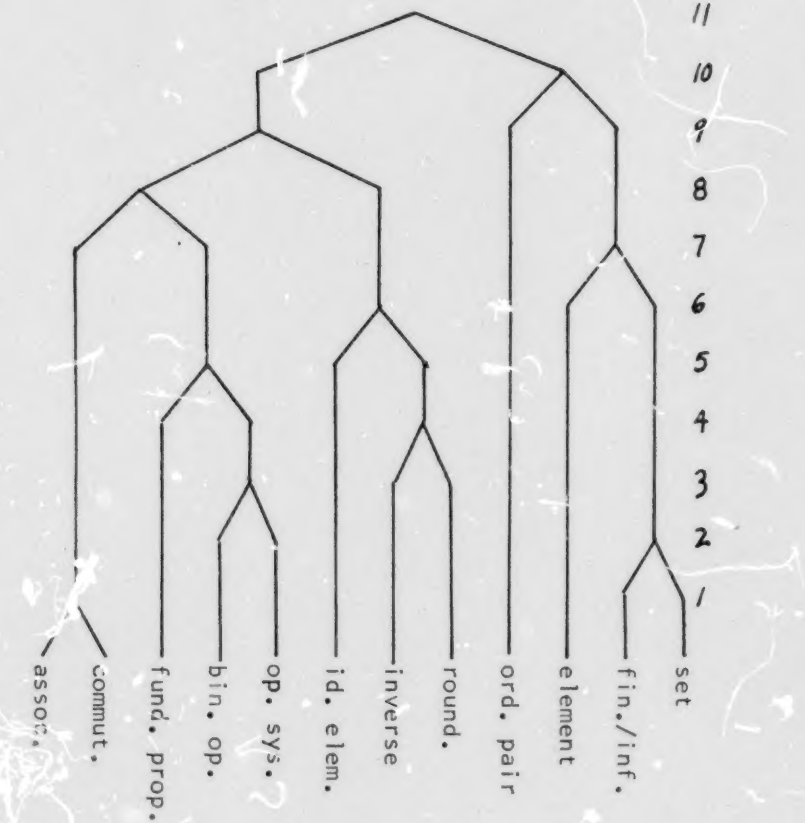
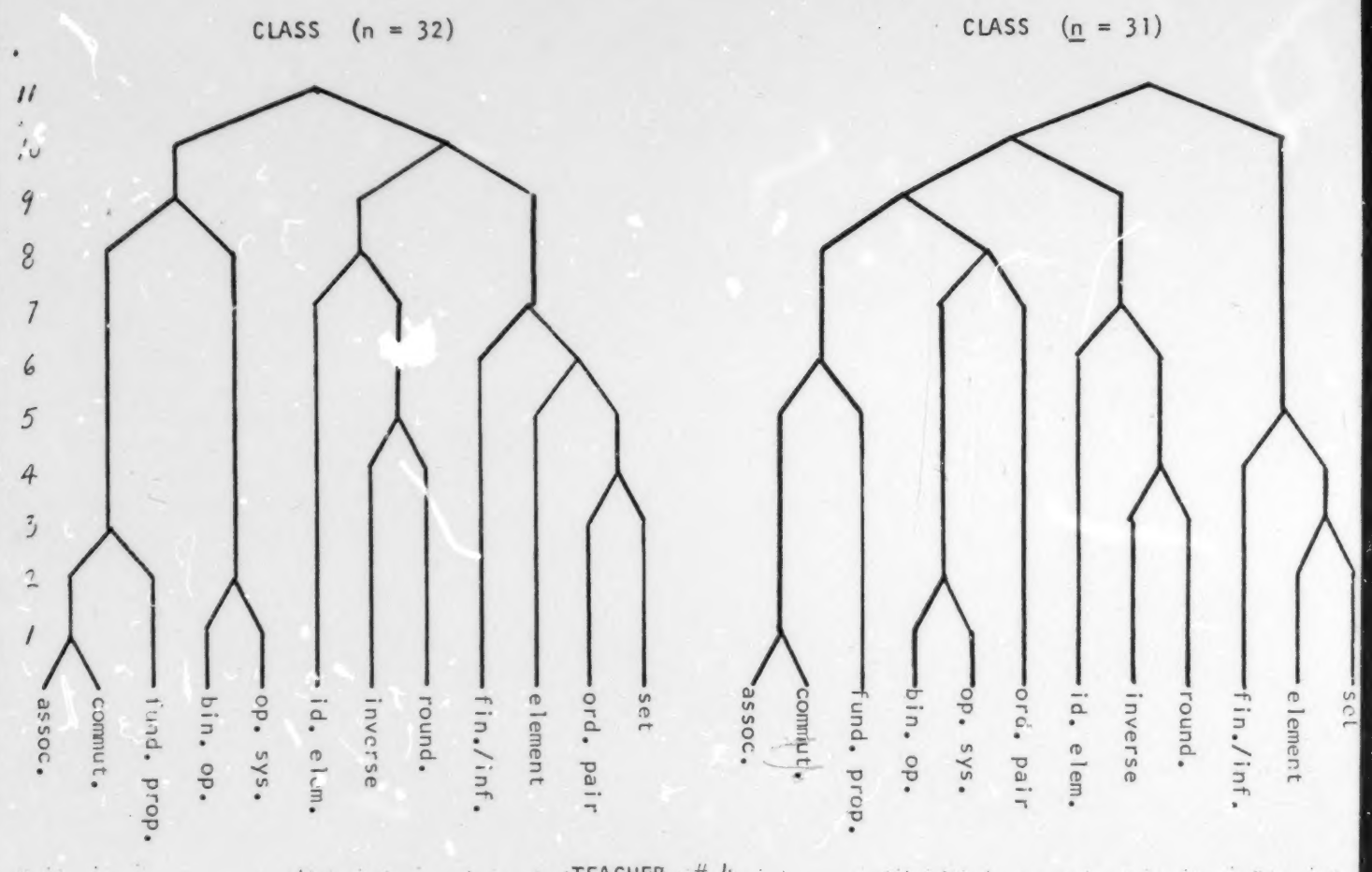


Figure 10.



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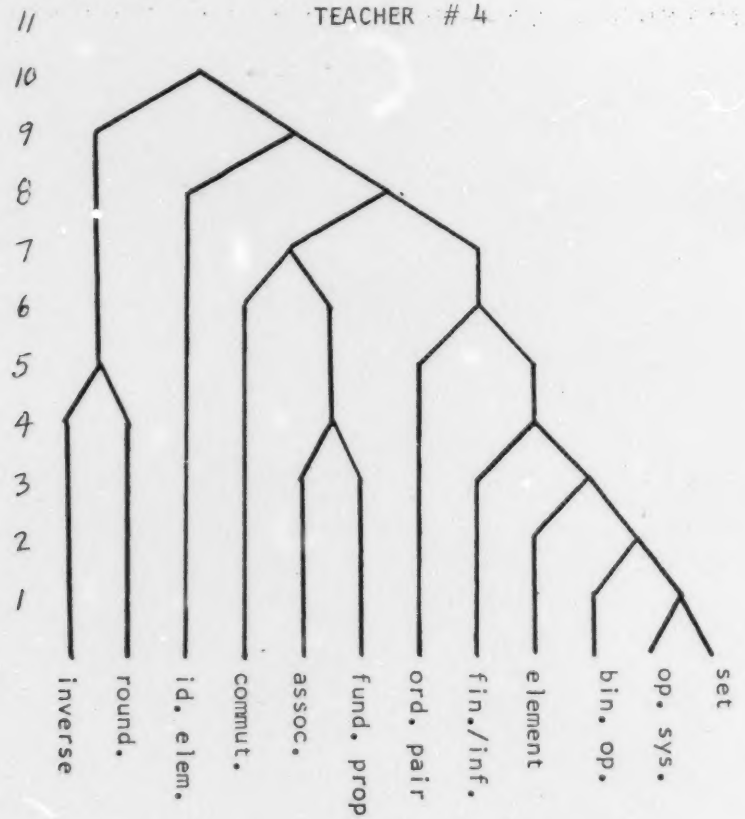
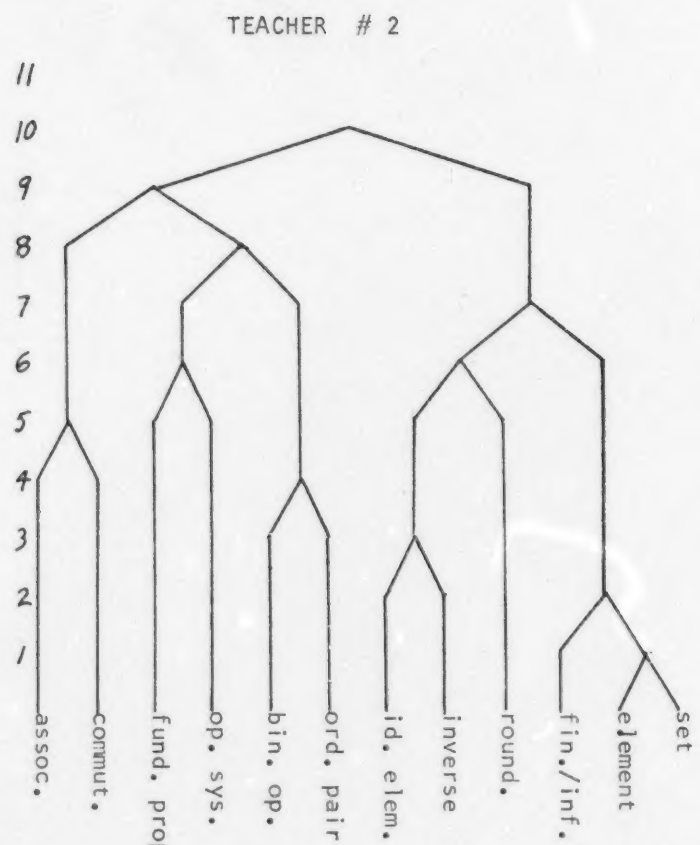


Figure 11.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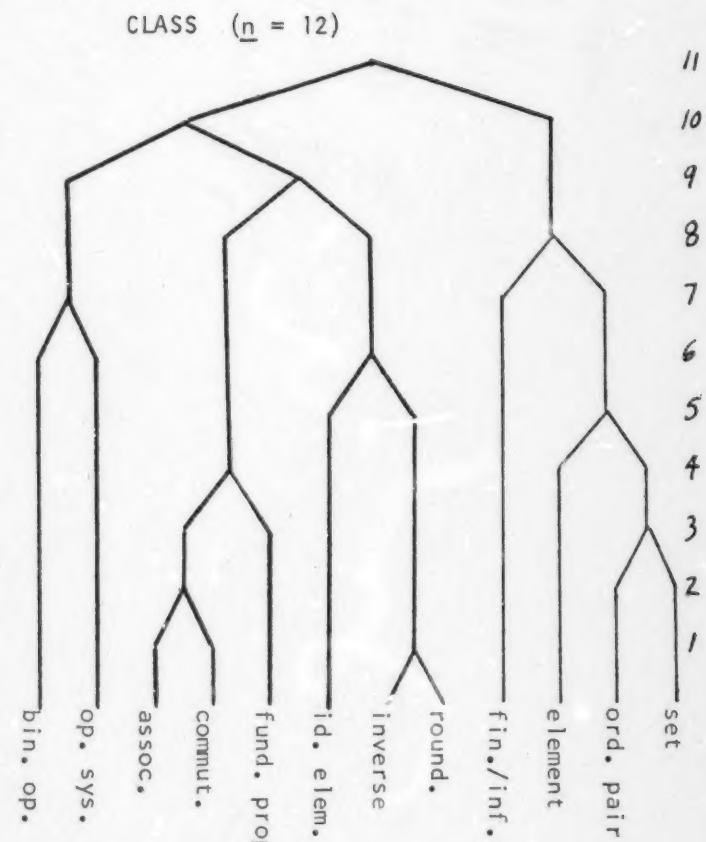
CLASS (n = 12)
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assoc.
commut.
fund. prop.
id. elem.
inverse
round.
fin./inf.
element
ord. pair
set

Figure 12.